Lecture 9: Financial Frictions and Amplification

Macroeconomics EC2B1
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In these lecture notes we will construct a theoretical model showing how adverse shocks to an economy may be amplified by worsening financial market conditions. This possibility was first formalized in two influential articles: Ben Bernanke and Mark Gertler (1989), "Agency Costs, Net Worth, and Business Fluctuations" and Nobu Kiyotaki and John Moore (1997), "Credit Cycles". The Kiyotaki-Moore paper was written while both were faculty at LSE (John Moore still has a part-time appointment `https://www.lse.ac.uk/economics/people/faculty/john-moore`). So this is very much an "LSE idea"!

The main idea can be described as follows: borrowing is limited by collateral. Collateral takes the form of an asset and therefore the amount of borrowing depends on the price of the asset. The price of the asset in turn depends on demand of consumers or firms, which in turn depends on the amount of borrowing. This can lead to a feedback loop as in Figure 1. We

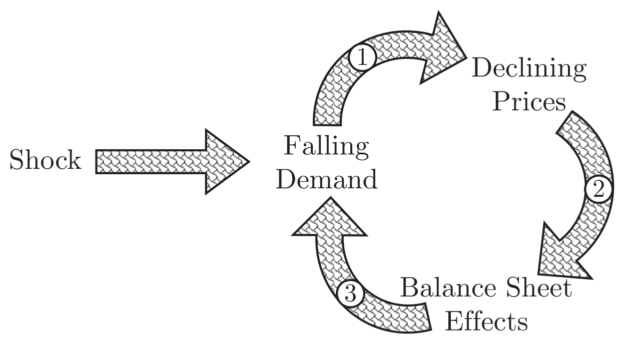

Figure 1: Financial Amplification

will refer to this mechanism as "Financial Amplification". You will sometimes here other terms like "financial accelerator," "debt deflation," or "Fisherian deflation." All of these describe the same basic idea. The model in these lecture notes is a two-period version of a paper by Olivier Jeanne and Anton Korinek (2012), "Managing Credit Booms and Busts: A Pigouvian Taxation Approach."

The model we really want to get to is in section 3 of these lecture notes. Before we get there, sections 1 and 2 will separately cover the two main building blocks of the model: borrowing and saving in a small open economy, and some consumption based asset pricing theory.

1 Preliminaries: Borrowing and Saving in a Small Open Economy

Consider a small open economy populated by many identical households. What distinguishes a small open economy from a large (closed or open) economy is that (some of) the prices faced by the households are determined in the rest of the world and hence do not respond to the choices of the households. In our setup, the only price will be the interest rate, and the assumption is that the household can borrow and lend at a fixed world interest rate r^*. The justification is that the world is populated by many households (and firms) characterized by some preferences (and technology) and that our small open economy only corresponds to an infinitesimally small slice of the world (technically it is a country with population size N and we take the limit as $N \to 0$).

Consumers in our small open economy solve the problem:

$$\begin{aligned} \max_{c_1,c_2,d_1} \; & u(c_1) + \beta u(c_2) \quad \text{s.t.} \\ & c_1 = y_1 + d_1 \\ & c_2 + d_1(1+r^*) = y_2 \\ & d_1 \leq \kappa y_1, \qquad \kappa \geq 0 \end{aligned}$$

Here, c_1 and c_2 are consumption as usual. y_1 and y_2 are incomes in the two time period. For simplicity, income takes the form of an unearned endowment. Production could be incorporated at the expense of some extra notation. d_1 is debt, that is how much the household borrows in period 1 and repays in period 2 at the fixed world interest rate r^*. Put differently and to relate it to our earlier lecture notes, d_1 is negative bond holdings.[1] If $d_1 < 0$, it means that the households saves/lends. The last equation is a *borrowing constraint*. It says that consumers can only borrow up to a fraction κ of their first period income y_1 (or up to a multiple of their income when $\kappa > 1$. There are different justifications for such a borrowing constraint. One is *limited commitment*. That is, a household can refuse to repay its debt in which debtors can confiscate a fraction κ of his income. κ therefore parametrizes the quality of credit markets. If

[1] Alternatively, we could have written the problem as

$$\begin{aligned} \max_{c_1,c_2,d_1} \; & u(c_1) + \beta u(c_2) \quad \text{s.t.} \\ & c_1 + b_1 = y_1 \\ & c_2 = y_2 + b_1(1+r^*) \\ & -b_1 \leq \kappa y_1, \qquad 0 \leq \kappa \leq 1 \end{aligned}$$

The two are equivalent with $b_1 = -d_1$. I find the formulation above slightly more intuitive.

$\kappa = \infty$, the constraint never binds and credit markets are perfect. If $\kappa = 0$ households cannot borrow at all.

No Borrowing Constraint Suppose for the moment that there is no borrowing constraint (or equivalently, $\kappa = \infty$ so that it never binds). In that case, the problem is exactly the same as in our earlier lecture notes and we know how to solve it. First combine the budget constraints into a single intertemporal budget constraints:

$$c_1 + \frac{c_2}{1+r^*} = y_1 + \frac{y_2}{1+r^*} \equiv y^{PDV} \tag{1}$$

Next, take first order conditions, in particular derive the *Euler equation*:

$$u'(c_1) = \beta(1+r^*)u'(c_2) \tag{2}$$

Given that the world interest rate is fixed, this problem is easily solved: (1) and (2) are two equations in two unknowns, c_1 and c_2. To make things even simpler, we typically make the following assumption in small open economy models.

Assumption 1: The world interest rate r^* satisfies

$$\beta(1+r^*) = 1.$$

From the Euler equation (2), Assumption 1 immediately implies that households choose a flat consumption profile:

$$u'(c_1) = u'(c_2) \quad \Rightarrow \quad c_1 = c_2$$

Plugging back into budget constraint (1) we get

$$c_1^u = c_2^u = \frac{1+r^*}{2+r^*} y^{PDV}$$

where u superscripts stand for "unconstrained." Finally, we can back out the amount of borrowing necessary to achieve this consumption allocation:

$$d_1^u = c_1^u - y_1 = \frac{y_2 - y_1}{2+r^*}.$$

As expected, because $\beta(1+r^*) = 1$, the households borrows whenever $y_2 > y_1$ and saves whenever $y_2 < y_1$. We will use (c_1^u, c_2^u, d_1^u) as a benchmark in the case with a borrowing constraint below. Note also that (c_1^u, c_2^u, d_1^u) only depend on parameters of the model (y_1, y_2, r^*).

Borrowing Constraint. Now consider the case where the borrowing constraint $d_1 \leq \kappa y_1$ is present (and where $\kappa < \infty$). There are two cases:

Case 1: $d_1^u \leq \kappa y_1$ (loose constraint). The household can obtain the unconstrained allocation (c_1^u, c_2^u, d_1^u) by borrowing less than is allowed by the borrowing constraint. Therefore, this will also be the optimal choice in the presence of the constraint and the constraint will never bind. Everything is as if there were no constraint in the first place.

Case 2: $d_1^u > \kappa y_1$ (binding constraint). The household *cannot* obtain the unconstrained allocation (c_1^u, c_2^u, d_1^u). This is because borrowing would have to be more than allowed by the borrowing constraint. Given this, the household will borrow as much as it can $d_1 = \kappa y_1$ and its consumption choice will be:

$$c_1 = (1+\kappa)y_1, \quad c_2 = y_2 - \kappa y_1(1+r^*)$$

Note that $c_1 < c_1^u = c_2^u < c_2$. That is, households can no longer smooth consumption perfectly. This implies that the borrowing constraint makes them strictly worse off. More precisely, their welfare satisfies:

$$W = u(c_1) + \beta u(c_1) < u(c_1^u) + \beta u(c_2^u) = W^u.$$

A Credit Crunch: as a warm-up exercise for below, consider a "credit crunch" by which we mean an exogenous tightening of the collateral constraint as captured by a decline in the parameter κ. This results in a decline in first-period consumption c_1 and a rise in second-period consumption c_2. The household is unambiguously worse off, i.e. W falls, because he can smooth consumption even less than before.

2 Preliminaries: Consumption Based Asset Pricing

The second building block of our model of financial amplification will be a simple theory of equilibrium determination of the price of an asset. In particular, we will use what is called consumption-based asset pricing theory or the "Lucas Asset Pricing Model" after Robert E. Lucas Jr. (1978), "Asset Pricing in an Exchange Economy."

Consider our economy from before but without borrowing and lending. Instead there is an asset a_t, for example houses, that the household can invest in. The household can buy the asset at price p_1 in period 1, and the asset pays a dividend D in period 2, i.e. this is the cashflow the asset owner gets from owning the asset (e.g. the revenues from renting the house). Importantly, the asset is in *fixed supply*: $a_0^s = a_1^s = 1$ (so perhaps land is the better example than houses). The question, we will try to answer is: how is the asset priced in equilibrium? Note that this is the polar opposite from the usual question we ask, which would be: how much of the asset trades in equilibrium? Here we already know that $a_0 = a_1 = 1$ in equilibrium, but we the question is what the price of the asset is.

Households solve:

$$\max_{c_1, c_2, a_1} u(c_1) + \beta u(c_2) \quad \text{s.t.}$$
$$c_1 + p_1 a_1 = y_1 + p_1 a_0$$
$$c_2 = y_2 + D a_1$$

That is, the household is born with some assets a_0 (perhaps inherited from his parents). It can then choose to purchase some additional assets $a_1 - a_0$ at price p_1.[2]. The total amount of assets it carries over to period 2, a_1, then pays the dividend (cashflow) D. The dividend and unearned income constitute consumption tomorrow. Note we have assumed that the asset pays no dividend in the first period. This is actually without loss of generality because we can always subsume first period dividends into first period income.[3] Furthermore, the asset cannot be sold in the second period (there is no $p_2 a_1$ term in the second period budget constraint). This is because ours is a two period model: the world ends after period two so noone would want to buy the asset.

The model can be solved as usual. Set up the Lagrangean and take first order conditions. This gives rise to a first order condition which can be written as:

$$p_1 u'(c_1) = \beta D u'(c_2)$$

Intuitively, the left hand side is the marginal cost and the right hand side the marginal benefit from buying the asset: an extra unit of the asset (an extra square foot of the house) costs $\$p_1$ which results in a utility loss of $\$p_1 u'(c_1)$; on the other hand, the asset pays $\$D$ tomorrow which results in a utility gain of $\$Du'(c_2)$ discounted at rate β. Alternatively, we can also define the asset's return $R = D/p_1$ and write this equation as $u'(c_1) = \beta R u'(c_2)$. This way of writing things makes clear that this is just a standard Euler equation.

[2]The budget constraint can also be written in terms of *net* purchases of the asset $c_1 + p_1(a_1 - a_0) = y_1$, which may be more intuitive.

[3]That is, write the budget constraint as $c_1 + p_1 a_1 = \tilde{y}_1 + p_1 a_0$ where $\tilde{y}_1 = y_1 + D_1 a_0$.

In equilibrium $a_0 = a_1 = 1$. Hence $c_1^e = y_1, c_2^e = y_2 + D$ ("e" superscripts denote "equilibrium"). Therefore, the price of the asset satisfies:

$$p_1 = \frac{\beta u'(c_2^e)}{u'(c_1^e)} D = \frac{\beta u'(y_2 + D)}{u'(y_1)} D \tag{3}$$

This answers our question: what is the equilibrium price of the asset? We will therefore refer to (3) as the "asset pricing equation." Note again that this is the opposite of the usual thought experiment. Rather than asking "given prices, what is consumption?" we asked "given consumption, what is the price?"

Example: log utility, $u(c) = \log c$. In this case:

$$p_1 = \frac{\beta c_1^e}{c_2^e} D$$

Intuitively, the price of the asset is high if the dividend D is high. Perhaps a little less intuitively, the price of the asset also depends on consumption in the two time periods. This is because the household not only uses the asset as an investment, i.e. to get the dividend D, but also to smooth consumption (recall there are no other savings opportunities in this economy). So for example if income in period 1, y_1, is low and hence consumption c_1 is also low, the asset price p_1 is low. This is because households really want to borrow rather than to invest in the asset (why would you want to buy a house if you're starving?). This decreases demand and hence the asset price.

That the consumption allocation matters for asset prices is the distinctive feature of consumption-based asset pricing theories. That low consumption in the first period leads to a decline in the asset price will also be an important feature of our model of amplification below.

Generalization: Infinite Horizon As an aside, the model can be generalized to an infinite horizon as follows:

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_t) \quad \text{s.t.}$$

$$c_t + p_t a_{t+1} = y_t + D_t a_t + p_t a_t$$

Following the usual strategy, one can show that the asset pricing equation is:

$$p_t = \frac{\beta u'(c_{t+1})}{u'(c_t)} (D_{t+1} + p_{t+1}) \tag{4}$$

In contrast to (3) in the two period model, the price of the asset today also depends on the price of the asset tomorrow, p_{t+1}. This is because the asset can be resold tomorrow. This equation is starting to look very much like the equation $p_t = y_t + \beta p_{t+1}$ from the lecture notes on bubbles. In particular, if utility functions are linear so that individuals are risk neutral, $u(c) = c$, then

$$p_t = \beta(D_{t+1} + p_{t+1}).$$

Defininig $y_t = \beta D_{t+1}$, we obtain the same equation as (1) in lecture notes (16) (there are some differences in timing assumptions). The difference between risk-neutral asset pricing and consumption-based asset pricing is that in the latter marginal utilities of consumption matter for asset prices. Or put differently, consumption-based theories use a different discount factor m_t rather than β:

$$p_t = m_t(D_{t+1} + p_{t+1}), \quad m_t \equiv \frac{\beta u'(c_{t+1})}{u'(c_t)}.$$

The term m_t is often called the "stochastic discount factor".[4] It is the discount factor β, "corrected" for marginal utilities to reflect the fact that individuals may be risk averse or have a motif for consumption smoothing.

Finally note that we can also write equation (4) as

$$u'(c_t) = \beta R_t u'(c_{t+1}) \quad \text{where} \quad R_t = \frac{D_{t+1} + p_{t+1}}{p_t}$$

is the asset's (gross) return which is the sum of the asset's dividend yield D_{t+1}/p_t plus capital gains p_{t+1}/p_t. This way of writing things makes clear that this is just a standard Euler equation.

3 Borrowing and Saving with a "Collateral Asset": Financial Amplification

Now, we will put everything together and, by combining elements from sections 1 and 2, we will show how financial amplification effects may arise. The key in the entire story will be a borrowing constraint in which the amount of debt d_1 a household can take out is constrained

[4] Obviously in our setting without uncertainty this terminology doesn't make that much sense. Instead, consider the further generalization where the dividend D_t is a random variable. In that case the asset pricing formula is

$$p_t = \mathbb{E}_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} (D_{t+1} + p_{t+1}) \right]$$

where $\mathbb{E}_t$ is the expectation taken with the time t information set.

by the value of its assets. There are two ways of writing this borrowing constraint which are slightly different. The first one is:

$$d_1 \leq \kappa p_1 a_0. \tag{5}$$

I will refer to this formulation as the "houses as ATMs formulation". The idea is that the household uses as collateral the value of its *existing* assets a_0. That is, the household can use its house as collateral to take out a loan to finance its consumption, hence the "houses as ATMs" terminology. See the 2011 article by Atif Mian and Amir Sufi, "House Prices, Home Equity-Based Borrowing, and the U.S. Household Leverage Crisis" and this Economist article `https://www.economist.com/finance-and-economics/2009/09/03/withdrawal-symptoms?story_id=14365068` for more discussion. We will return to this below.

The second formulation is:

$$d_1 \leq \kappa p_1 a_1. \tag{6}$$

I will refer to this formulation as the "mortgage formulation." The idea is that now the household can use as collateral the value of its *future* assets a_1, that is assets it does not actually own at the moment it takes out the loan. Or put differently, the household finances part of the asset with a loan and then uses the asset as a security to guarantee he will pay it back. This is just like a mortgage: you buy a house which costs you $p_1 a_1$, and you can borrow up to a fraction κ of the purchase price; the remaining $(1-\kappa)p_1 a_1$ is your down payment.

The two formulations are similar and both lead to financial amplification effects, but there are some subtle differences.

3.1 Equilibrium with Houses as ATM Formulation (5)

Households solve:

$$\begin{aligned} \max_{c_1, c_2, a_1, d_1} \quad & u(c_1) + \beta u(c_2) \quad \text{s.t.} \\ & c_1 + p_1 a_1 = y_1 + p_1 a_0 + d_1 \\ & c_2 + d_1(1 + r^*) = y_2 + D a_1 \\ & d_1 \leq \kappa p_1 a_0 \end{aligned}$$

And the asset is still in fixed supply: $a_0^s = a_1^s = 1$. Let's first consider the case where there is no borrowing constraint.

No Borrowing Constraint. **Claim:** The equilibrium is characterized by an Euler equation and an asset pricing equation

$$u'(c_1) = \beta(1+r^*)u'(c_2) \tag{7}$$

$$p_1 = \frac{\beta u'(c_2)}{u'(c_1)} D \tag{8}$$

Assume $\beta(1+r^*) = 1$. Then the solution is

$$c_1 = c_2, \qquad p_1 = \beta D$$

Again for reference, let us denote by c_1^u, c_2^u, d_1^u and p_1^u the "unconstrained" consumption, debt and equilibrium asset price.

Borrowing Constraint. There are again two cases. Also note that the equilibrium borrowing limit is $\kappa p_1 a_0 = \kappa p_1$ where we use that $a_0 = 1$ in equilibrium.

Case 1: $d_1^u \leq \kappa p_1^u$ (loose constraint). As before the equilibrium outcome is the same as if the constraint were not present.

Case 2: $d_1^u > \kappa p_1^u$ (binding constraint). Now things are more complicated. As in section 1, households borrow all the way to the constraint. Hence their debt is $d_1 = \kappa p_1$ and from the budget constraint, equilibrium consumption is

$$c_1 = y_1 + \kappa p_1, \quad c_2 = y_2 + D - \kappa p_1(1+r^*)$$

Substituting into the asset pricing equation (8), p_1 is therefore determined by the equation

$$p_1 = \frac{\beta u'(y_2 + D - \kappa p_1(1+r^*))}{u'(y_1 + \kappa p_1)} D$$

This equation implicitly determines the price p_1. In general, this is a bit of a nasty equation. The reason is that p_1 appears in three places. With log utility, $u(c) = \log c$, we obtain

$$p_1 = \frac{\beta(y_1 + \kappa p_1)}{y_2 + D - \kappa p_1(1+r^*)} D$$

which is a quadratic in p_1. This can be solved but I instead find it easier to make the following simplifying assumption.

Assumption 2 (log-linear utility): The period utility functions are different in the first and second periods. In particular, utility is logarithmic in the first period and linear in the second period:

$$u_1(c_1) + \beta u_2(c_2), \quad u_1(c_1) = \log c_1, \quad u_2(c_2) = c_2$$

Let us also make one more assumption whose role will become clear shortly:

Assumption 3: $\kappa\beta D < 1$.

It is easy to show that with the utility function in Assumption 2, the asset pricing equation (8) becomes

$$p_1 = \frac{\beta u_2'(c_2)}{u_1'(c_1)} D = \beta D c_1 \tag{9}$$

Using that $c_1 = y_1 + \kappa p_1$, we can solve for the equilibrium asset price

$$p_1 = \frac{\beta D y_1}{1 - \beta D \kappa} \tag{10}$$

and equilibrium consumption

$$c_1 = \frac{y_1}{1 - \beta D \kappa} \tag{11}$$

Now we are done. Note that Assumption 3 ensures that the equilibrium c_1 and p_1 are finite and positive. More on this momentarily. Importantly, consumption and the asset price feature *financial amplification*. To see this, consider a negative exogenous shock to first period income (in a more general model, this could be triggered by a negative aggregate TFP shock which translates into lower wages and hence lower income y_1 – we will consider other shocks momentarily). The impact of this shock on consumption is

$$\frac{\partial c_1}{\partial y_1} = \frac{1}{1 - \beta D \kappa} > 1.$$

Contrast, this with an economy with *complete financial autarky*, i.e. no borrowing and lending so that $c_1 = y_1$. In that case consumption would always move one by one with income. In our economy with financial amplification however, not only do households not smooth consumption. Instead, their consumption is even more volatile than income!

The reason for this is is exactly the mechanism in Figure 1. The main two equations to see this are the expression for consumption from the budget constraint and the equation determining the price of the asset as a function of consumption (9):

$$c_1 = y_1 + \kappa p_1 \tag{12}$$

$$p_1 = \beta D c_1 \tag{13}$$

Consider now a negative shock that leads to a decline by \$1 of y_1. Naturally from (14) this has a direct effect: a decline in consumption by the same amount. In turn, this has an effect on the asset price. From (15), the drop in c_1 leads to a drop in p_1 by \$$\beta D$ (the demand for houses dropped depressing house prices). Without financial amplification, this would be the end of the story. But it is not because there is a feedback through the collateral constraint: how much households can borrow depends on the price of the asset and since the price of the asset dropped, borrowing and hence consumption drops further by a total of \$$\kappa\beta D$ (see (14)). But this in turn leads to a further drop in the asset price, leading to further consumption drops and so on.

Mathematically, we can denote each iteration in this loop by $n = 0, 1, 2...$ and write

$$c_1^n = y_1 + \kappa p_1^n \tag{14}$$

$$p_1^n = \beta D c_1^{n+1} \tag{15}$$

Substituting the second equation into the first, we have

$$c_1^n = y_1 + \kappa\beta D c_1^{n+1}$$

Solving forward, we have

$$c_1^0 = y_1 \sum_{n=0}^{\infty} (\kappa\beta D)^n = \frac{1}{1 - \kappa\beta D} y_1.$$

Note the role of Assumption 3: $\kappa\beta D < 1$. Its purpose is to ensure that this process doesn't "blow up." If the assumption were violated financial amplification is so powerful that an equilibrium ceases to exist!

Also other shocks can trigger this "diabolic loop." For example, consider a shock to future cashflows D. We have that

$$\frac{\partial c_1}{\partial D} = \frac{\kappa\beta y_1}{(1 - \kappa\beta D)^2}$$

Note that since $\kappa\beta D < 1$, $(1 - \kappa\beta D)^2$ is a really small number and so $\kappa\beta y_1/(1 - \kappa\beta D)^2$ is a huge number. So financial amplification is even more extreme in this case.

House Prices and Household Leverage During the Great Recession. See the two papers by Mian and Sufi "House Prices, Home Equity-Based Borrowing, and the U.S. Household Leverage Crisis" (2011) and "What Explains the 2007–2009 Drop in Employment?" (2014). Mian and Sufi argue that a mechanism similar to the one described in these notes may have played an important role during the Great Recession. In particular, before the recession optimistic expectations (say about the cashflows from owning a house) triggered an increase in house prices. This led to a build up of household leverage. But then the recession hit and expectations got more pessimistic (e.g. leading to a decline in expected D) and this forced households to delever. This translated into households cutting their spending considerably, which in turn led to high unemployment. Mian and Sufi try to establish this mechanism by looking across counties within the United States. Figure 2 is one of their main figures. It shows

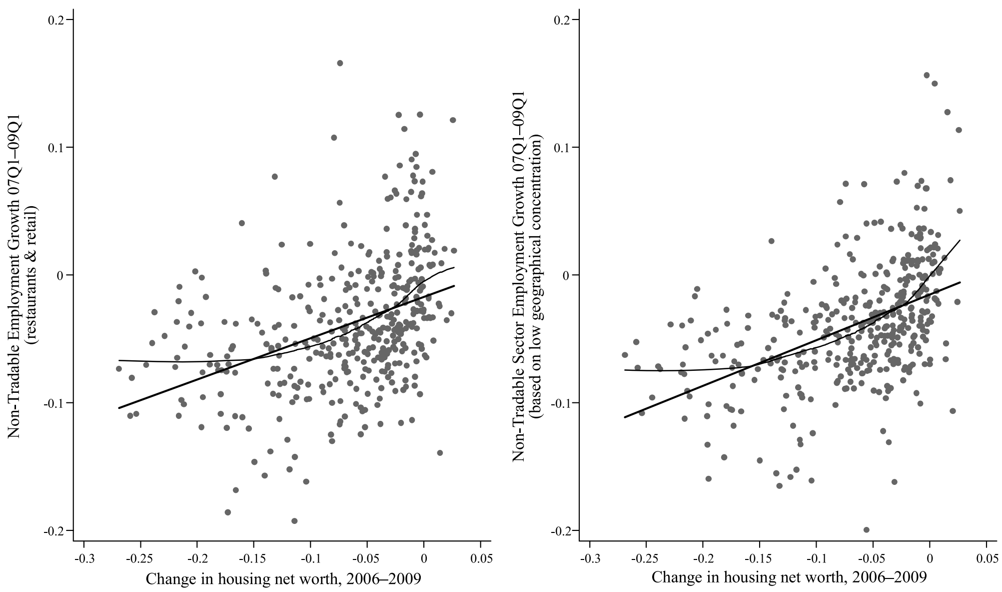

FIGURE 1.—Non-tradable employment and the housing net worth shock. This figure presents scatter-plots of county-level non-tradable employment growth from 2007Q1 to 2009Q1 against the change in housing net worth from 2006 to 2009. The left panel defines industries in restaurant and retail sector as non-tradable, and the right panel defines industries as non-tradable if they are geographically dispersed throughout the United States. The sample includes counties with more than 50,000 households. The thin black line in the left panel is the non-parametric plot of non–tradable employment growth against change in housing net worth.

Figure 2: from Mian and Sufi (2014)

that the recession was most severe (employment fell the most from 2007 to 2009) in counties which suffered the largest declines in housing net worth.

3.2 Mortgage Formulation (6)

Now households solve:

$$
\begin{aligned}
\max_{c_1,c_2,a_1,d_1} \quad & u(c_1)+\beta u(c_2) \quad \text{s.t.} \\
& c_1 + p_1 a_1 = y_1 + p_1 a_0 + d_1 \\
& c_2 + d_1(1+r^*) = y_2 + D a_1 \\
& d_1 \leq \kappa p_1 a_1
\end{aligned}
$$

Good exercise: solving for this economy's equilibrium allocation.